Chapter 3

ch03 homework

- Vector addition + subtraction; components
- Scalar product, derivatives
- Cross product.

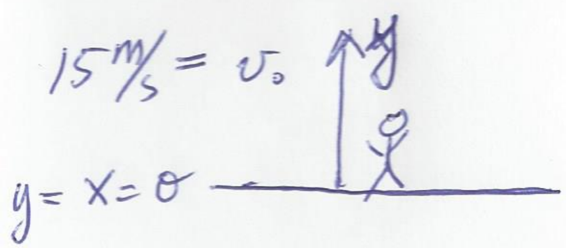
Ch 3b homework.

- Projectile motion
- Relative velocity
- Relativistic velocity

Kinematic equations:

$$\begin{cases} x = x_0 + v_0 t + \frac{1}{2} a t^2 \\ v(t) = v_0 + a t \\ v^2 = v_0^2 + 2a\underbrace{(x - x_0)}_{\Delta x} \end{cases}$$

$$g = 9.80 \frac{m}{s^2} = 32 \, ft/s^2$$



- What is the highest point?
- What is the velocity after 5 s?
- What is the velocity when the ball reaches a location 15 m below the original point?

$$y = 0 + v_0 t - \frac{1}{2} g t^2$$

$$v = v_0 - g t$$

Highest point:

$0 = v_0^2 - 2g\,y_{max}$; $0 = (15\,m/s)^2 - 19.6\,\frac{m}{s^2}\,y_{max}$

$y_{max} = \frac{225}{19.6}\,m = \underline{\underline{11.5\,m}}$

$v = 15\frac{m}{s} - 9.8 \cdot 5\,\frac{m}{s}$; $v = -34\,\frac{m}{s}$

$v^2 = \cancel{a}\;15^2 - 19.6\,(-15\,m)$

$= 225 + 294$

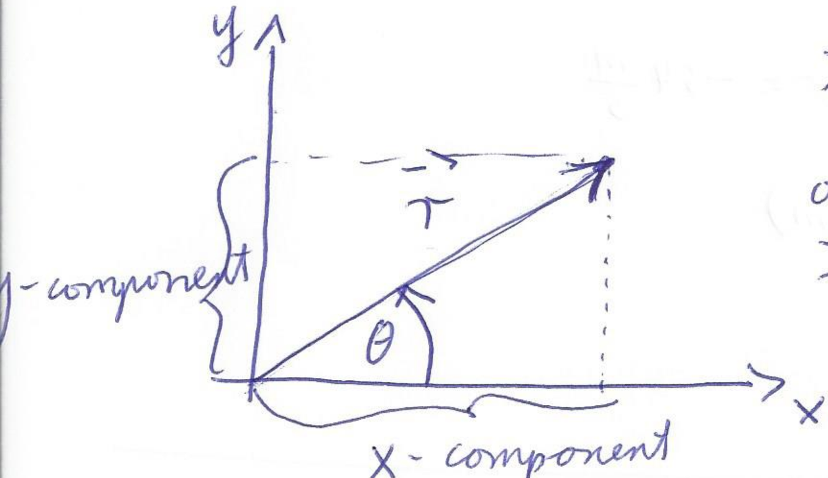
$v = \pm 22.8\,\frac{m}{s}$

chapter 3

Onweb:

$\boxed{\text{vectors}}$: physical quantities which have a magnitude and a direction.

location vector: $\vec{r}$



cartesian coordinates

$\vec{r} = \langle x, y \rangle$

ordered pair of numbers.

$\vec{r} = \langle x, y, z \rangle$

example: $x = 5 \quad y = 3$

$$r = \sqrt{x^2 + y^2} = \sqrt{25 + 9}$$
$$= \sqrt{34}$$

$\tan\theta = \frac{y}{x} = \frac{3}{5} \quad \theta = 31°$

$\vec{r} = \langle \sqrt{34}, 31° \rangle$ polar coordinates

Unit vectors have the length 1.

x : direction $\vec{i}$

y : " $\vec{j}$

z : " $\vec{k}$

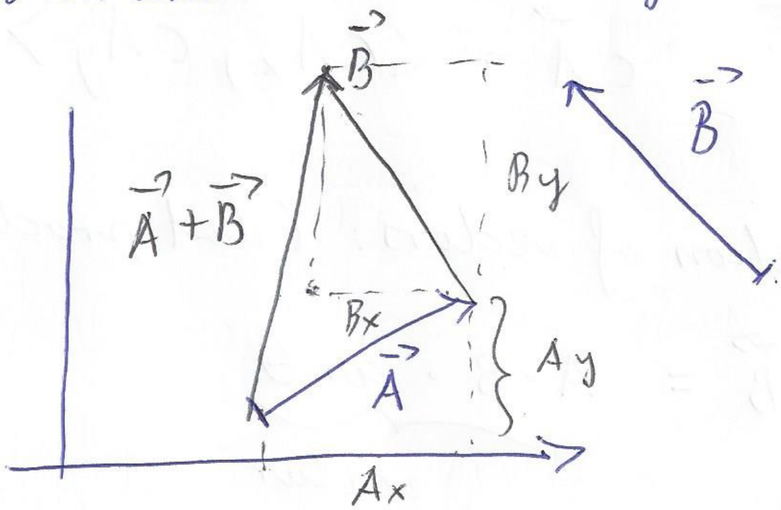
$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$

$$\vec{r} = x\vec{i} + y\vec{j} + z \cdot \vec{k} = \langle x, y, z \rangle$$

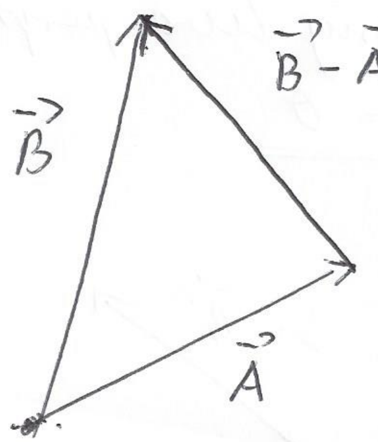
Addition and subtraction of vectors.



$\vec{A} = \langle A_x, A_y \rangle \qquad \vec{B} = \langle B_x, B_y \rangle$

$\vec{A} + \vec{B} = \langle A_x + B_x, A_y + B_y \rangle$

Subtraction



$\vec{A} + \vec{B} - \vec{A} - \vec{B} = \vec{0}$

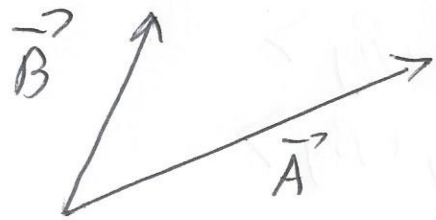
$\vec{0} = \langle 0, 0, 0 \rangle$

Multiplication:

by a scalar: c $\qquad c\vec{A} = \langle cA_x, cA_y \rangle$

Scalar multiplication of vectors: (dot product)

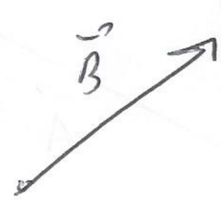
$$\vec{A} \cdot \vec{B} = \underbrace{A \cdot B \cdot \cos\theta}_{\text{scalar}}$$

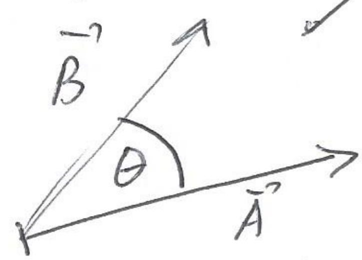


$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The scalar product of any two perpendicular vectors is 0. $\underline{\cos 90^\circ = 0}$

Example: $\vec{A} = \langle 5, 3 \rangle$
$\vec{B} = \langle 6, 9 \rangle$



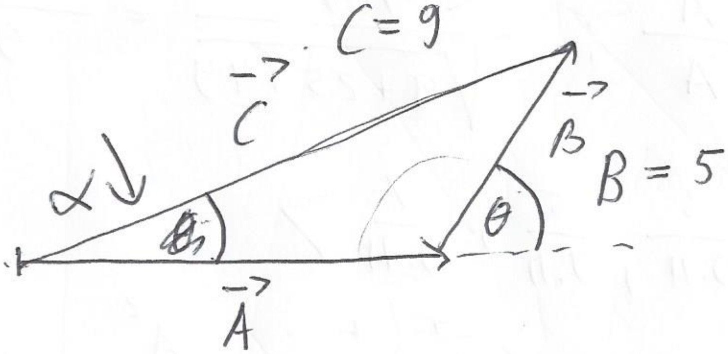


$A = \sqrt{25+9} \qquad B = \sqrt{36+81}\ ; \quad A \cdot B = 63.07$

$63.07 \cdot \cos\theta = 5 \cdot 6 + 3 \cdot 9 = 30 + 27 = 57$

57

p 6



$A = 6$

$$\vec{A} + \vec{B} = \vec{C} \quad \Big|^2$$

$$(\vec{A} + \vec{B})(\vec{A} + \vec{B}) = (\vec{A} + \vec{B})^2 = \vec{C}^2$$

$$\vec{A}^2 + 2\vec{A}\cdot\vec{B} + \vec{B}^2 = \vec{C}^2$$

$$A^2 + B^2 + 2A\cdot B\cdot\cos\theta = C^2$$

$$\vec{A} - \vec{C} = -\vec{B} \quad \Big|^2$$

$$A^2 + C^2 - 2\vec{A}\cdot\vec{C} = B^2$$

$$-2A\cdot C\cdot\cos\alpha = B^2 - A^2 - C^2$$

$$2AC\cos\alpha = A^2 + C^2 - B^2$$

$$\cos\alpha = \frac{A^2 + C^2 - B^2}{2AC}$$

Find a unit vector in the direction of $\vec{A} = \langle 3,5,7 \rangle$

$$\vec{U}_{\vec{A}} = \frac{\vec{A}}{A} \; ; \; \frac{\langle 3,5,7 \rangle}{\sqrt{9+25+49}} = \frac{\langle 3,5,7 \rangle}{9.11}$$

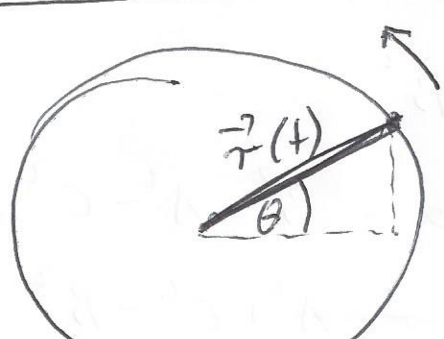
$$= \left\langle \frac{3}{9.11}, \frac{5}{9.11}, \frac{7}{9.11} \right\rangle$$

$$\left(\vec{U}_A\right)^2 = 1 = \left(\frac{\vec{A}}{A}\right)^2 = \frac{A^2}{A^2} = 1$$

$$\vec{A} = \langle 3t^2, 5t, 5 \rangle$$

$$\frac{d\vec{A}}{dt} = \langle 6t, 5, 0 \rangle$$

Unit vectors in circular motion:



$$\vec{r}(t) = r \cdot \vec{U}_r$$

$$\vec{U}_r = \langle 1 \cdot \cos\theta, 1 \cdot \sin\theta \rangle$$

$$|\vec{U}_r| = \sqrt{\cos^2\theta + \sin^2\theta}$$
$$= 1$$

magnitude

θ is a function of time

... velocity $= 2\pi r$ ..., T = period angularvelocity (constant) $\frac{2\pi}{T} = \omega$ omega

$$\omega = \frac{\theta}{t} \qquad \theta = \omega \cdot t$$

$$\vec{U}_r = \langle \cos\omega t, \sin\omega t \rangle$$

$$\vec{r} = r \langle \cos\omega t, \sin\omega t \rangle$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r \langle -\sin\omega t \cdot \omega, \cos\omega t \cdot \omega \rangle$$

$$= r\omega \underbrace{\langle -\sin\omega t, \cos\omega t \rangle}_{\vec{U}_\theta}$$

$$\vec{U}_r \cdot \vec{U}_\theta = \langle \cos\omega t, \sin\omega t \rangle \langle -\sin\omega t, \cos\omega t \rangle$$

$$= -\cos\omega t \cdot \sin\omega t + \sin\omega t \cos\omega t = 0$$

$$\vec{U}_\theta \perp \vec{U}_r$$